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Super-Jordanian $\mathcal{U}_\mathsf{h}(osp(1|2))$ algebra as a contraction limit

B Abdesselam[1,2], R Chakrabarti[3], A Hazzab[4] and A Yanallah[2]

[1] Laboratoire de Physique Théorique d'Oran (LPTO), Université d'Oran Es-Sénia, 31100-Oran, Algérie
[2] Laboratoire de Physique Quantique de la Matière et Modélisations Mathématiques (LPQ3M)[5], Centre Universitaire de Mascara, 29000-Mascara, Algérie
[3] Department of Theoretical Physics, University of Madras, Guindy Campus, Madras 600025, India
[4] Laboratoire de Modélisation et Méthode de Calcul, Centre Universitaire de Saida, 20000-Saida, Algérie

E-mail: boucif@yahoo.fr and ranabir@imsc.res.in

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Abstract
We develop a generic representation-independent contraction procedure for obtaining, for instance, R_h matrices and L operators of arbitrary dimensions for the quantized super-Jordanian $\mathcal{U}_\mathsf{h}(osp(1|2))$ algebra from the pertinent quantities of the standard q-deformed $\mathcal{U}_q(osp(1|2))$ algebra.

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1. Introduction

Quantum deformations of the Lie superalgebra $osp(1|2)$ have been studied extensively [1–7] both from the point of view of investigating integrable physical models, and also because of their intrinsic mathematical importance. Distinct bialgebra structures existing on the classical $osp(1|2)$ superalgebra have been studied [4]. The Lie superalgebra $osp(1|2)$ has three even $(h, b_\pm)$ and two odd (e, f) generators, which obey the commutation relations

$$\begin{array}{llll} [h,e]=e & [h,f]=-f & \{e,f\}=-h & \\ [h,b_\pm]=\pm 2b_\pm & [b_+,b_-]=h & & \\ [b_+,f]=e & [b_-,e]=f & b_+=e^2 & b_-=-f^2. \end{array} \tag{1.1}$$

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The classical r-matrix containing an odd generator

$$r = h \wedge b_{+} - e \wedge e \tag{1.2}$$

has recently been quantized [5] using nonlinear basis elements. The corresponding quantized super-Jordanian $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra is known [5, 6] to satisfy the triangularity condition. Various differential geometric structures on the noncommutative super-Jordanian $\mathrm{Fun}_{\mathsf{h}}(OSp(1|2))$ function algebra have been constructed [7]. An important issue observed [3] in this context is that the quantum R_{h} matrix of the $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra in the fundamental representation may be obtained *via* a contraction mechanism from the corresponding R_q matrix of the standard q-deformed $\mathcal{U}_q(osp(1|2))$ algebra in the $q \to 1$ limit. A generalization of this contraction procedure for arbitrary representations, though clearly desirable as it will allow us to systematically obtain various quantities of interest of the $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra from the corresponding quantities of the $\mathcal{U}_q(osp(1|2))$ algebra, has not been achieved so far.

On the other hand, a generic technic developed earlier [8–10] allowed us to extract the quantum R and $\mathbf{T}$ matrices of arbitrary representations of the Jordanian $\mathcal{U}_{\mathsf{h}}(sl(N))$ algebra from the corresponding operators of the q-deformed $\mathcal{U}_q(sl(N))$ algebra. A suitable adaptation of this procedure is used in section 2 to obtain the quantum R_{h} matrices of arbitrary representations, and the L operator of the super-Jordanian $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra. The L operator obtained here immediately produces, *via* the standard FRT [11] procedure, the Hopf structure of the Borel subalgebra of the $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra. This agrees with the known result [5] proving the validity of our contraction scheme. Our method may be readily used to derive the quantum $\mathbf{T}_{\mathsf{h}}$ matrices of arbitrary representations of the $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra.

2. Contraction process

The Hopf structure of the super-Jordanian $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra using nonlinear basis elements was obtained [5] previously. For comparing with our subsequent results we list it, with a slightly altered normalization, as

$$\begin{gathered}
[H, E] = \frac{1}{2}(T + T^{-1})E \qquad [H, F] = -\frac{1}{4}(T + T^{-1})F - \frac{1}{4}F(T + T^{-1}) \\
\{E, F\} = -H \qquad [H, T^{\pm 1}] = T^{\pm 2} - 1 \\
[H, Y] = -\frac{1}{2}(T + T^{-1})Y - \frac{1}{2}Y(T + T^{-1}) - \frac{\mathsf{h}}{4}E(T - T^{-1})F - \frac{\mathsf{h}}{4}F(T - T^{-1})E \\
[T^{\pm 1}, Y] = \pm\frac{\mathsf{h}}{2}(T^{\pm 1}H + HT^{\pm 1}) \qquad E^2 = \frac{T - T^{-1}}{2\mathsf{h}} \qquad F^2 = -Y \\
[T^{\pm 1}, F] = \pm\mathsf{h}T^{\pm 1}E \qquad [Y, E] = \frac{1}{4}(T + T^{-1})F + \frac{1}{4}F(T + T^{-1}) \\
\Delta(H) = H \otimes T^{-1} + T \otimes H + \mathsf{h}ET^{1/2} \otimes ET^{-1/2} \qquad \Delta(E) = E \otimes T^{-1/2} + T^{1/2} \otimes E \\
\Delta(F) = F \otimes T^{-1/2} + T^{1/2} \otimes F \qquad \Delta(T^{\pm 1}) = T^{\pm 1} \otimes T^{\pm 1} \\
\Delta(Y) = Y \otimes T^{-1} + T \otimes Y + \frac{\mathsf{h}}{2}ET^{1/2} \otimes T^{-1/2}F + \frac{\mathsf{h}}{2}T^{1/2}F \otimes ET^{-1/2} \\
\varepsilon(H) = \varepsilon(E) = \varepsilon(F) = \varepsilon(Y) = 0 \qquad \varepsilon(T^{\pm 1}) = 1 \\
S(H) = -H - \mathsf{h}E^2 \qquad S(E) = -E \qquad S(F) = -F + \frac{\mathsf{h}}{2}E \\
S(T^{\pm 1}) = T^{\mp 1} \qquad S(Y) = -Y + \frac{\mathsf{h}}{2}H + \frac{\mathsf{h}^2}{4}E^2
\end{gathered} \tag{2.1}$$

where h is the deformation parameter. The $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra has only *one* Borel subalgebra generated by the elements $(H, E, T^{\pm 1})$. Kulish observed [3] that the R_{h} matrix in the fundamental representation of the super-Jordanian $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra may be obtained *via* a transformation, singular in the $q \to 1$ limit, from the corresponding R_q matrix in the fundamental representation of the standard q-deformed $\mathcal{U}_q(osp(1|2))$ algebra.

Our task is to generalize the above contraction procedure for arbitrary representations. As an application of our contraction scheme, we construct the L operator corresponding to the Borel subalgebra of the super-Jordanian $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra from the corresponding L operator of the standard q-deformed $\mathcal{U}_q(osp(1|2))$ algebra. To this end, we first quote some well-known [1, 2] results on the $\mathcal{U}_q(osp(1|2))$ algebra. The $\mathcal{U}_q(osp(1|2))$ algebra is generated by three elements $(\hat{h}, \hat{e}, \hat{f})$ obeying the Hopf structure

$$\begin{aligned}
&[\hat{h}, \hat{e}] = \hat{e} \qquad [\hat{h}, \hat{f}] = -\hat{f} \qquad \{\hat{e}, \hat{f}\} = -[h]_q \\
&\Delta(\hat{h}) = \hat{h} \otimes 1 + 1 \otimes \hat{h} \qquad \Delta(\hat{e}) = \hat{e} \otimes q^{-\hat{h}/2} + q^{\hat{h}/2} \otimes \hat{e} \qquad \Delta(\hat{f}) = \hat{f} \otimes q^{-\hat{h}/2} + q^{\hat{h}/2} \otimes \hat{f} \\
&\varepsilon(\hat{h}) = \varepsilon(\hat{e}) = \varepsilon(\hat{f}) = 0 \qquad S(\hat{h}) = -\hat{h} \qquad S(\hat{e}) = -q^{-1/2}\hat{e} \qquad S(\hat{f}) = -q^{1/2}\hat{f}
\end{aligned} \tag{2.2}$$

where $[x]_q = (q^x - q^{-x})/(q - q^{-1})$. To facilitate our later application, we choose the $(4j+1)$-dimensional irreducible representation of the $\mathcal{U}_q(osp(1|2))$ algebra in an asymmetrical manner as follows:

$$\begin{aligned}
&\hat{h}|jm\rangle = 2m|jm\rangle \qquad \hat{e}|jm\rangle = |jm+1/2\rangle \qquad \hat{e}|jj\rangle = 0 \\
&\hat{f}|jm\rangle = -[j+m]_q[[j-m+1/2]]_q|jm-1/2\rangle \qquad \text{for} \quad j-m \text{ integer} \\
&\qquad = [[j+m]]_q[j-m+1/2]_q|jm-1/2\rangle \qquad \text{for} \quad j-m \text{ half-integer}
\end{aligned} \tag{2.3}$$

where $[[x]]_q = (q^x - (-1)^{2x}q^{-x})/(q^{1/2} + q^{-1/2}), 2j \in \mathsf{N}, m = j, j-1/2, \ldots, -(j-1/2), -j$.

Following the strategy adopted earlier [8–10] for constructing the Jordanian deformation of the $sl(N)$ algebra, we give here the general recipe for obtaining the quantum $R_{\mathsf{h}}^{j_1;j_2}$ matrix of an arbitrary representation of the $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra. An explicit demonstration is given for the $1/2 \otimes j$ representation. The relevant $R_{\mathsf{h}}^{1/2;j}$ matrix may be directly interpreted as the L operator corresponding to the Borel subalgebra of the $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra. Our construction may, obviously, be generalized for an arbitrary $j_1 \otimes j_2$ representation. The primary ingredient for our method is the $R_q^{1/2;j}$ matrix [1] of the $\mathcal{U}_q(osp(1|2))$ algebra in the $1/2 \otimes j$ representation. A suitable similarity transformation is performed on this $R_q^{1/2;j}$ matrix. The transforming matrix is singular in the $q \to 1$ limit. *For the transformed matrix, the singularities, however, systematically cancel yielding a well-defined construction*. The transformed object, finite in the $q \to 1$ limit, directly furnishes the $R_{\mathsf{h}}^{1/2;j}$ matrix for the super-Jordanian $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra. Interpreting, as mentioned above, the $R_{\mathsf{h}}^{1/2;j}$ matrix obtained here as the L operator corresponding to the Borel subalgebra of the $\mathcal{U}_{\mathsf{h}}(osp(1|2))$ algebra, we use the standard FRT procedure [11] for constructing the full Hopf structure of the said Borel subalgebra.

The $R_q^{1/2;j}$ matrix of the tensored $1/2 \otimes j$ representation of the $\mathcal{U}_q(osp(1|2))$ algebra may be obtained from [1] as

$$R_q^{\frac{1}{2};j} = \begin{pmatrix} q^{\hat{h}} & -\omega q^{\hat{h}/2}\hat{f} & -\omega(1+q^{-1})\hat{f}^2 \\ 0 & 1 & \omega q^{-(\hat{h}+1)/2}\hat{f} \\ 0 & 0 & q^{-\hat{h}} \end{pmatrix} \tag{2.4}$$

where $\omega = q - q^{-1}$. We now introduce a transforming matrix M, singular in the $q \to 1$ limit, as

$$M = E_{q^2}(\eta \hat{e}^2) \tag{2.5}$$

where

$$E_{q^2}(x) = \sum_{n=0}^{\infty} \frac{x^n}{[n]_{q^2}!} \qquad \eta = \frac{\mathrm{h}}{q^2 - 1}. \tag{2.6}$$

For any finite value of j the series (2.6) may be terminated after setting $\hat{e}^{4j+1} = 0$. As the transforming operator M in (2.5) depends only on the generator $\hat{e}$, our subsequent results assume the *simplest* form for the asymmetric choice of the representation (2.3). Our contraction scheme, however, remains valid independent of the choice of representation. The $R_q^{j_1;j_2}$ matrix of the $\mathcal{U}_q(osp(1|2))$ algebra may now be subjected to a similarity transformation followed by a limiting process:

$$\tilde{R}_{\mathrm{h}}^{j_1;j_2} \equiv \lim_{q\to 1} \left[\left(M_{j_1}^{-1} \otimes M_{j_2}^{-1}\right) R_q^{j_1;j_2} \left(M_{j_1} \otimes M_{j_2}\right)\right]. \tag{2.7}$$

In the following we will present explicit results for the operator $\tilde{R}_{\mathrm{h}}^{1/2;j}$. In our calculation a class of operators

$$\mathcal{T}_{(\alpha)} = (E_{q^2}(\eta \hat{e}^2))^{-1} E_{q^2}(q^{2\alpha}\eta \hat{e}^2) \tag{2.8}$$

satisfying

$$\mathcal{T}_{(\alpha+\beta)} q^{(\alpha+\beta)\hat{h}} = \mathcal{T}_{(\alpha)} q^{\alpha\hat{h}} \mathcal{T}_{(\beta)} q^{\beta\hat{h}} \tag{2.9}$$

play an important role. To evaluate the $q \to 1$ limiting value of the operator $\mathcal{T}_{(\alpha)}$, we use the identity

$$\mathcal{T}_{(1)} - \mathcal{T}_{(-1)} = \eta(q^2 - q^{-2})\, \hat{e}^2. \tag{2.10}$$

Evaluating term by term, the limiting values of $\mathcal{T}_{(\pm 1)}|_{q\to 1} (\equiv \tilde{T}_{(\pm 1)})$ are found to be *finite*; and for these finite operators the product structure (2.9) yields $\tilde{T}_{(\pm\alpha)} = (\tilde{T}_{(\pm 1)})^{\alpha}$, where $\tilde{T}_{(\alpha)} = \lim_{q\to 1} \mathcal{T}_{(\alpha)}$. Writing $\tilde{T}_{(\pm 1)} = \tilde{T}^{\pm 1}$ henceforth, we immediately observe that in the $q \to 1$ limit, the identity (2.10) assumes the form

$$\tilde{T} - \tilde{T}^{-1} = 2\mathrm{h}e^2 \quad \Longrightarrow \quad \tilde{T}^{\pm 1} = \pm \mathrm{h}e^2 + \sqrt{1 + \mathrm{h}^2 e^4}. \tag{2.11}$$

This is our crucial result. Two other operator identities playing key roles are listed below:

$$\hat{f}\hat{e}^{2n} = \hat{e}^{2n}\hat{f} - \frac{q}{q+1}\{n\}_{q^2}\hat{e}^{2n-1}\hat{t} - \frac{1}{q+1}\{n\}_{q^{-2}}\hat{e}^{2n-1}\hat{t}^{-1} \tag{2.12}$$

$$\begin{aligned} \hat{f}^2\hat{e}^{2n} = {} & \hat{e}^{2n}\hat{f}^2 + q\frac{q-1}{q+1}\{n\}_{q^2}\hat{e}^{2n-1}\hat{t}\hat{f} - q^{-1}\frac{q-1}{q+1}\{n\}_{q^{-2}}\hat{e}^{2n-1}\hat{t}^{-1}\hat{f} \\ & + \frac{q}{q+1}\left(\frac{1}{\omega}\{n\}_{q^4} - q^2\frac{q-1}{q+1}\begin{Bmatrix} n \\ 2 \end{Bmatrix}_{q^2}\right)\hat{e}^{2(n-1)}\hat{t}^2 \\ & - \frac{1}{q+1}\left(\frac{1}{\omega}\{n\}_{q^{-4}} - q^{-2}\frac{q-1}{q+1}\begin{Bmatrix} n \\ 2 \end{Bmatrix}_{q^{-2}}\right)\hat{e}^{2(n-1)}\hat{t}^{-2} \\ & - \frac{q}{(q+1)^3}(q\{n\}_{q^2} + \{n\}_{q^{-2}})\hat{e}^{2(n-1)} \end{aligned} \tag{2.13}$$

where $\{x\}_q = \frac{1-q^x}{1-q}$, $\{n\}_q! = \{n\}_q\{n-1\}_q \cdots \{1\}_q$, $\{0\}_q! = 1$, $\begin{Bmatrix} n \\ m \end{Bmatrix}_q = \frac{\{n\}_q!}{\{n-m\}_q!\{m\}_q!}$ and $\hat{t}^{\pm 1} = q^{\pm\hat{h}}$. Using the above identities systematically and passing to the limit $q \to 1$, it follows in a representation-independent way that in our construction of the operator $\tilde{R}_{\mathrm{h}}^{1/2;j}$ *via* (2.7), all singularities *cancel* yielding a well-defined answer

$$\tilde{R}_{\mathrm{h}}^{\frac{1}{2};j} = \begin{pmatrix} \tilde{T} & \mathrm{h}\tilde{T}^{\frac{1}{2}}e & -\mathrm{h}\tilde{H} + \frac{\mathrm{h}}{4}(\tilde{T} - \tilde{T}^{-1}) \\ 0 & 1 & -\mathrm{h}\tilde{T}^{-\frac{1}{2}}e \\ 0 & 0 & \tilde{T}^{-1} \end{pmatrix} \tag{2.14}$$

where $\tilde{H} = \frac{1}{2}(\tilde{T} + \tilde{T}^{-1})h = \sqrt{1 + \mathrm{h}^2 e^4}\,h$. One way of interpreting (2.14) is to consider it a recipe for obtaining the finite-dimensional R_{h} matrices of the $\mathcal{U}_{\mathrm{h}}(osp(1|2))$ algebra. For instance, using the classical $j = 1$ representation, obtained from (2.3) in the $q \to 1$ limit, we obtain the $R_{\mathrm{h}}^{1/2;1}\big(=\tilde{R}_{\mathrm{h}}^{1/2;1}\big)$ matrix as follows:

$$R_{\mathrm{h}}^{\frac{1}{2};1} = \begin{pmatrix}
1 & 0 & \mathrm{h} & 0 & \frac{\mathrm{h}^2}{2} & 0 & \mathrm{h} & 0 & \frac{\mathrm{h}^2}{2} & 0 & -2\mathrm{h} & 0 & \frac{\mathrm{h}^2}{2} & 0 & \mathrm{h}^3 \\
0 & 1 & 0 & \mathrm{h} & 0 & 0 & 0 & \mathrm{h} & 0 & \frac{\mathrm{h}^2}{2} & 0 & -\mathrm{h} & 0 & \frac{\mathrm{h}^2}{2} & 0 \\
0 & 0 & 1 & 0 & \mathrm{h} & 0 & 0 & 0 & \mathrm{h} & 0 & 0 & 0 & 0 & 0 & \frac{\mathrm{h}^2}{2} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \mathrm{h} & 0 & 0 & 0 & \mathrm{h} & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mathrm{h} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\mathrm{h} & 0 & \frac{\mathrm{h}^2}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\mathrm{h} & 0 & \frac{\mathrm{h}^2}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\mathrm{h} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\mathrm{h} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\mathrm{h} & 0 & \frac{\mathrm{h}^2}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\mathrm{h} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\mathrm{h} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}. \tag{2.15}$$

Thus our contraction procedure allows us to construct quantum R_{h} matrices of arbitrary dimensions from the corresponding R_q matrices.

The matrix (2.14) may also be interpreted as the L operator of the $\mathcal{U}_{\mathrm{h}}(osp(1|2))$ algebra. To this end, we first use the following invertible map of the quantum $\mathcal{U}_{\mathrm{h}}(osp(1|2))$ algebra (2.1) on the classical algebra (1.1):

$$E = e \qquad H = \tilde{H} \qquad F = f + \frac{\mathrm{h}}{4}\left(\frac{\tilde{T} - 1}{\tilde{T} + 1}\right)e - \frac{\mathrm{h}}{2}\left(\frac{\tilde{T} - 1}{\tilde{T} + 1}\right)eh \qquad T = \tilde{T} \qquad Y = F^2. \tag{2.16}$$

The map (2.16) satisfies the algebraic relations (2.1); and the corresponding twist operator may also be determined. Using the map (2.16) the operator (2.14) may be recast in terms of the *deformed generators* of the super-Jordanian $\mathcal{U}_{\mathrm{h}}(osp(1|2))$ algebra as

$$L \equiv \tilde{R}_{\mathrm{h}}^{\frac{1}{2};j} = \begin{pmatrix} T & \mathrm{h}T^{\frac{1}{2}}E & -\mathrm{h}H + \frac{\mathrm{h}}{4}(T - T^{-1}) \\ 0 & 1 & -\mathrm{h}T^{-\frac{1}{2}}E \\ 0 & 0 & T^{-1} \end{pmatrix}. \tag{2.17}$$

The above L operator allows immediate construction of the full Hopf structure of the Borel subalgebra of the $\mathcal{U}_{\mathrm{h}}(osp(1|2))$ algebra *via* the standard FRT formalism [11]. The algebraic relations for the generators $(H, E, T^{\pm 1})$ of the Borel subalgebra are given by

$$R_{\mathrm{h}}^{\frac{1}{2};\frac{1}{2}} L_1 L_2 = L_2 L_1 R_{\mathrm{h}}^{\frac{1}{2};\frac{1}{2}} \tag{2.18}$$

where the $\mathbf{Z}_2$ graded tensor product has been used in defining the operators: $L_1 = L \otimes \mathbf{I}$, $L_2 = \mathbf{I} \otimes L$. The coalgebraic properties of the said Borel subalgebra may be succinctly expressed as

$$\Delta(L) = L \dot{\otimes} L \qquad \varepsilon(L) = \mathbf{I} \qquad S(L) = L^{-1} \tag{2.19}$$

where L^{-1} is given by

$$L^{-1} = \begin{pmatrix} T^{-1} & -\mathsf{h}T^{-\frac{1}{2}}E & \mathsf{h}H + \frac{\mathsf{h}}{4}(T - T^{-1}) \\ 0 & 1 & \mathsf{h}T^{\frac{1}{2}}E \\ 0 & 0 & T \end{pmatrix}. \tag{2.20}$$

This completes our construction of the Hopf structure of the Borel subalgebra of the super-Jordanian $\mathcal{U}_\mathsf{h}(osp(1|2))$ algebra by employing the contraction scheme described earlier. Our results fully coincide with the Hopf structure given in (2.1). This validates our contraction scheme elaborated before. An ansatz for the L operator of the $\mathcal{U}_\mathsf{h}(osp(1|2))$ algebra was previously given in [3]. However, our method of obtaining the L operator of the $\mathcal{U}_\mathsf{h}(osp(1|2))$ algebra from the $R_q^{1/2;j}$ matrix (2.4) by using the contraction transformation discussed above was not observed in [3]. Our recipe (2.7) for obtaining the $R_\mathsf{h}^{j_1;j_2}$ matrix for a $j_1 \otimes j_2$ representation of the super-Jordanian $\mathcal{U}_\mathsf{h}(osp(1|2))$ algebra may be continued arbitrarily. The matrices such as $R_\mathsf{h}^{1;j}$ may be interpreted as higher-dimensional L operators [12] obeying duality relations with the relevant $\mathbf{T}$ matrices. An arbitrary $\mathbf{T}_\mathsf{h}^j$ matrix of the super-Jordanian function algebra $\mathrm{Fun}_\mathsf{h}(OSp(1|2))$ may be obtained from the corresponding standard $\mathbf{T}_q^j$ matrix of the $\mathrm{Fun}_q(OSp(1|2))$ algebra by using our contraction transformation

$$\mathbf{T}_\mathsf{h}^j = \lim_{q\to 1} \left[M_j^{-1}\mathbf{T}_q^j M_j\right]. \tag{2.21}$$

3. Conclusion

Generalizing the approach in [3], we, in this paper, have found a generic representation-independent way of extracting various structures such as arbitrary finite-dimensional R_h matrices and L operators of the super-Jordanian $\mathcal{U}_\mathsf{h}(osp(1|2))$ algebra from the corresponding quantities of the standard q-deformed $\mathcal{U}_q(osp(1|2))$ algebra. One way to understand our contraction process defined in (2.7) is that it projects out elements of *one* Borel subalgebra generated by (h, e) from the classical $\mathcal{U}(osp(1|2))$ algebra. The existence [5] of the invertible maps connecting $\mathcal{U}_\mathsf{h}(osp(1|2))$ and $\mathcal{U}(osp(1|2))$ algebras now provides a construction of the *triangular* super-Jordanian $\mathcal{U}_\mathsf{h}(osp(1|2))$ algebra from the q-deformed $\mathcal{U}_q(osp(1|2))$ algebra. Our approach may also be fruitfully used, for instance, to obtain the higher dimensional $\mathbf{T}_\mathsf{h}^j$ matrices. In particular, the noncommutative space covariant under the coaction of the $\mathbf{T}_\mathsf{h}^{(j=1)}$ matrix is of interest. This will be discussed elsewhere.

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